Sensitivity Analysis

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Sensitivity analysis (SA) is " the study of how the variation in the output of a model (numerical or otherwise) can be apportioned, qualitatively or quantitatively, to different sources of variation".[1] However, when the assumptions are uncertain, and/or there are alternative sets of assumptions to chose from, the inference will also be also uncertain. Investigating the uncertainty in the inference (regardless of its source) goes under the name of **Uncertainty analysis**.

Sensitivity Analysis tries to identify those assumptions which weight the most in determining the uncertainty in the inference ('screening' sensitivity analysis). 'Quantitative' sensitivity analysis tries not only to identify but also to quantify the relative importance the influential assumptions. In the preceding discussion the term 'factor' if often used instead of 'assumption' - implying that assumptions have been translated into factors entering the model, e.g. with defined numerical values possibly drawn from factor-value distributions - while 'model output' can be used instead of inference.

Overview

- A mathematical model is defined by a series of equations, input factors, parameters, and variables aimed to characterize the process being investigated.
- Input is subject to many sources of uncertainty including errors of measurement, absence of information and poor or partial understanding of the driving forces and mechanisms. This uncertainty imposes a limit on our confidence in the response or output of the model. Further, models may have to cope with the natural intrinsic variability of the system, such as the occurrence of stochastic events.

- Good modeling practice requires that the modeler provides an evaluation of the confidence in the model, possibly assessing the uncertainties associated with the modeling process and with the outcome of the model itself. Uncertainty and Sensitivity Analysis offer valid tools for characterizing the uncertainty associated with a model.
- In models involving many input variables sensitivity analysis is an essential ingredient of model building and quality assurance. National and international agencies involved in impact assessment studies have included section devoted to sensitivity analysis in their guidelines. Examples are the European Commission, the White House Office for Budget and Management, the Intergovernmental Panel on Climate Change and the US Environmental Protection Agency.

Methodology

• There are several possible procedures to perform uncertainty (UA) and sensitivity analysis (SA). The most common sensitivity analysis is sampling-based. A sampling-based sensitivity is one in which the model is executed repeatedly for combinations of values sampled from the distribution (assumed known) of the input factors. Sampling based methods can also be used to decompose the variance of the model output (see references).

… Methodology

• In general, UA and SA are performed jointly by executing the model repeatedly for combination of factor values sampled with some probability distribution. The following steps can be listed:

1. Specify the target function and select the input of interest
2. Assign a distribution function to the selected factors
3. Generate a matrix of inputs with that distribution(s) through an appropriate design
4. Evaluate the model and compute the distribution of the target function
5. Select a method for assessing the influence or relative importance of each input factor on the target function.

Applications

- Sensitivity Analysis can be used to determine:

1. The model resemblance with the process under study
2. The quality of model definition
3. Factors that mostly contribute to the output variability
4. The region in the space of input factors for which the model variation is maximum
5. Optimal - or instability - regions within the space of factors for use in a subsequent calibration study
6. Interactions between factors

Sensitivity Analysis is popular in financial applications, risk analysis, signal processing, neural networks and any area where models are developed. SA can also be used in model-based policy assessment studies see e.g. [1].

- **Environmental**

  - Computer environmental models are increasingly used in a wide variety of studies and applications. For example global climate model are used for both short term weather forecasts and long term climate change.
  - Moreover, computer models are increasingly used for environmental decision making at a local scale, for example for assessing the impact of a waste water treatment plant on a river flow, or for assessing the behavior and life length of bio-filters for contaminated waste water.

• In both cases sensitivity analysis may help understanding the contribution of the various sources of uncertainty to the model output uncertainty and system performance in general. In these cases, depending on model complexity, different sampling strategies may be advisable and traditional sensitivity indexes have to be generalized to cover multivariate sensitivity analysis, heteroskedastic effects and correlated inputs.

- **Business**

  - In a decision problem, the analyst may want to identify cost drivers as well as other quantities for which we need to acquire better knowledge in order to make an informed decision. On the other hand, some quantities have no influence on the predictions, so that we can save resources at no loss in accuracy by relaxing some of the conditions. See Corporate finance: Quantifying uncertainty.

Sensitivity analysis can help in a variety of other circumstances which can be handled by the settings illustrated below:

- to identify critical assumptions or compare alternative model structures
- guide future data collections
- detect important criteria
- optimize the tolerance of manufactured parts in terms of the uncertainty in the parameters
- optimize resources allocation
- model simplification or model lumping, etc.

However there are also some problems associated with sensitivity analysis in the business context:

• Variables are often interdependent, which makes examining them each individually unrealistic, e.g.: changing one factor such as sales volume, will most likely affect other factors such as the selling price.

• Often the assumptions upon which the analysis is based are made by using past experience/data which may not hold in the future.

• Assigning a maximum and minimum (or optimistic and pessimistic) value is open to subjective interpretation. For instance one persons 'optimistic' forecast may be more conservative than that of another person performing a different part of the analysis. This sort of subjectivity can adversely affect the accuracy and overall objectivity of the analysis.

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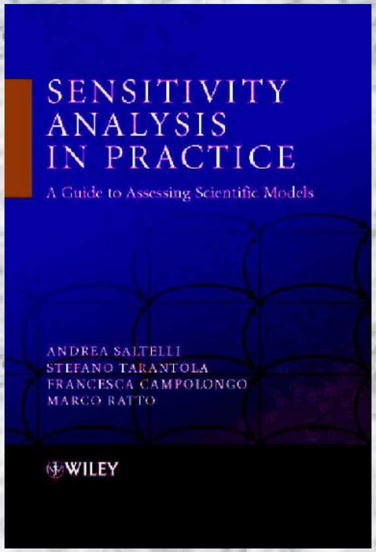
Tutorial on *Sensitivity Analysis*

To start with, we propose the following material :



- Cookbook
- Tutorial 1
- Tutorial 2
- Tutorial 3
- Book on Sensitivity Analysis (Preface)

A REAL TEST CASE

We also suggest some bibliographic material, where the reader will find the grater part of sensitivity analysis studies, together with applications and reviews:

- Sensitivity Analysis for Chemical Models
- Composite Indicators
- Archer, G.; Saltelli, A.; Sobol', I. M. *Journal of Statistical Computation and Simulation* **1997**, *58*, 99

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An Introduction to Sensitivity Analysis

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MIT System Dynamics in Education Project
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Dr. Jay W. Forrester

by
Lucia Breierova
Mark Choudhari
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Lemonade Stand

In the first exploration, let's look at a lemonade stand located on a college campus.

As usual, we are particularly interested in the behavior of the stock, the number of cups of lemonade that are ready to be sold to customers. The stand is open eight hours every day.

Howard, the owner, is the only person working in the stand.

Epidemics

In the second exploration we look at an epidemics model. The model was already used in a previous chapter in Road Maps, so it is possible that you have already built it.

Coffeehouse

We now return to Howard, the owner of the lemonade stand on a college campus.

Howard realized that it could be more profitable for him to sell coffee because students tend to drink more coffee than lemonade, and they drink it at any time of the day and night. Therefore, he closed his lemonade stand and opened a 24-hour Coffeehouse.

Howard bases the Coffeehouse model on the model he used in his lemonade stand to model the number of cups of "Coffee ready." We will run the simulation over a period of two days, or 48 hours.

Specific parameter values can change the appearance of the graphs representing the behavior of the system. But significant changes in behavior do not occur for all parameters. System dynamics models are in general insensitive to many parameter changes. It is the structure of the system, and not the parameter values, that has most influence on the behavior of the system.

Sensitivity analysis is an important tool in the model building process. By showing that the system does not react greatly to a change in a parameter value, it reduces the modeler's uncertainty in the behavior. In addition, it gives an opportunity for a better understanding of the dynamic behavior of the system.

We encourage you to experiment with the three models from this paper (as well as any other models that you have built) on your own. For example, try to change several parameters at the same time, observe the behavior produced, and compare it to the conclusions in this paper. Can you suggest any parameter values that would produce the "optimal," or most desirable behavior? The use of sensitivity analysis in such policy analysis will be explored in a later paper in this series.

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Keith C. Clarke, Department of Geology and Geography, Hunter College- City University of New York, and the City University of New York Graduate School and University Center.

Stacy Hoppen, Graduate Student, Department of Geology and Geography, Hunter College-City University of New York.

Leonard J. Gaydos, US Geological Survey, EROS Data Center, NASA¬Ames Research Center, Moffett Field, CA.

Sensitivity analysis

Sensitivity analysis (SA) is the study of how the variation (uncertainty) in the output of a mathematical model can be apportioned, qualitatively or quantitatively, to different sources of variation in the input of a model [1]. Put another way, it is a technique for systematically changing parameters in a model to determine the effects of such changes.

In more general terms uncertainty and sensitivity analyses investigate the robustness of a study when the study includes some form ofmathematical modelling. Sensitivity analysis can be useful to computer modellers for a range of purposes[2], including:

- support decision making or the development of recommendations for decision makers (e.g. testing the robustness of a result);
- enhancing communication from modellers to decision makers (e.g. by making recommendations more credible, understandable, compelling or persuasive);
- increased understanding or quantification of the system (e.g. understanding relationships between input and output variables); and
- model development (e.g. searching for errors in the model).

While uncertainty analysis studies the overall uncertainty in the conclusions of the study, sensitivity analysis tries to identify what source of uncertainty weights more on the study's conclusions. For example, several guidelines for modelling (see e.g. one from the US EPA) or forimpact assessment (see one from the European Commission) prescribe sensitivity analysis as a tool to ensure the quality of the modelling/assessment.

… Sensitivity analysis

The problem setting in sensitivity analysis has strong similarities with design of experiments. In design of experiments one studies the effect of some process or intervention (the 'treatment') on some objects (the 'experimental units'). In sensitivity analysis one looks at the effect of varying the inputs of a mathematical model on the output of the model itself. In both disciplines one strives to obtain information from the system with a minimum of physical or numerical experiments.

In uncertainty and sensitivity analysis there is a crucial trade off between how scrupulous an analyst is in exploring the input assumptions and how wide the resulting inference may be. The point is well illustrated by the econometrician Edward E. Leamer (1990) [3]:

I have proposed a form of organized sensitivity analysis that I call ‘global sensitivity analysis’ in which a neighborhood of alternative assumptions is selected and the corresponding interval of inferences is identified. Conclusions are judged to be sturdy only if the neighborhood of assumptions is wide enough to be credible and the corresponding interval of inferences is narrow enough to be useful.

Note Leamer’s emphasis is on the need for 'credibility' in the selection of assumptions. The easiest way to invalidate a model is to demonstrate that it is fragile with respect to the uncertainty in the assumptions or to show that its assumptions have not been taken 'wide enough'. The same concept is expressed by Jerome R. Ravetz, for whom bad modeling is when *uncertainties in inputs must be suppressed lest outputs become indeterminate.*[4]

In modern econometrics the use of sensitivity analysis to anticipate criticism is the subject of one of the ten commandments of applied econometrics (from Kennedy, 2007[5]):

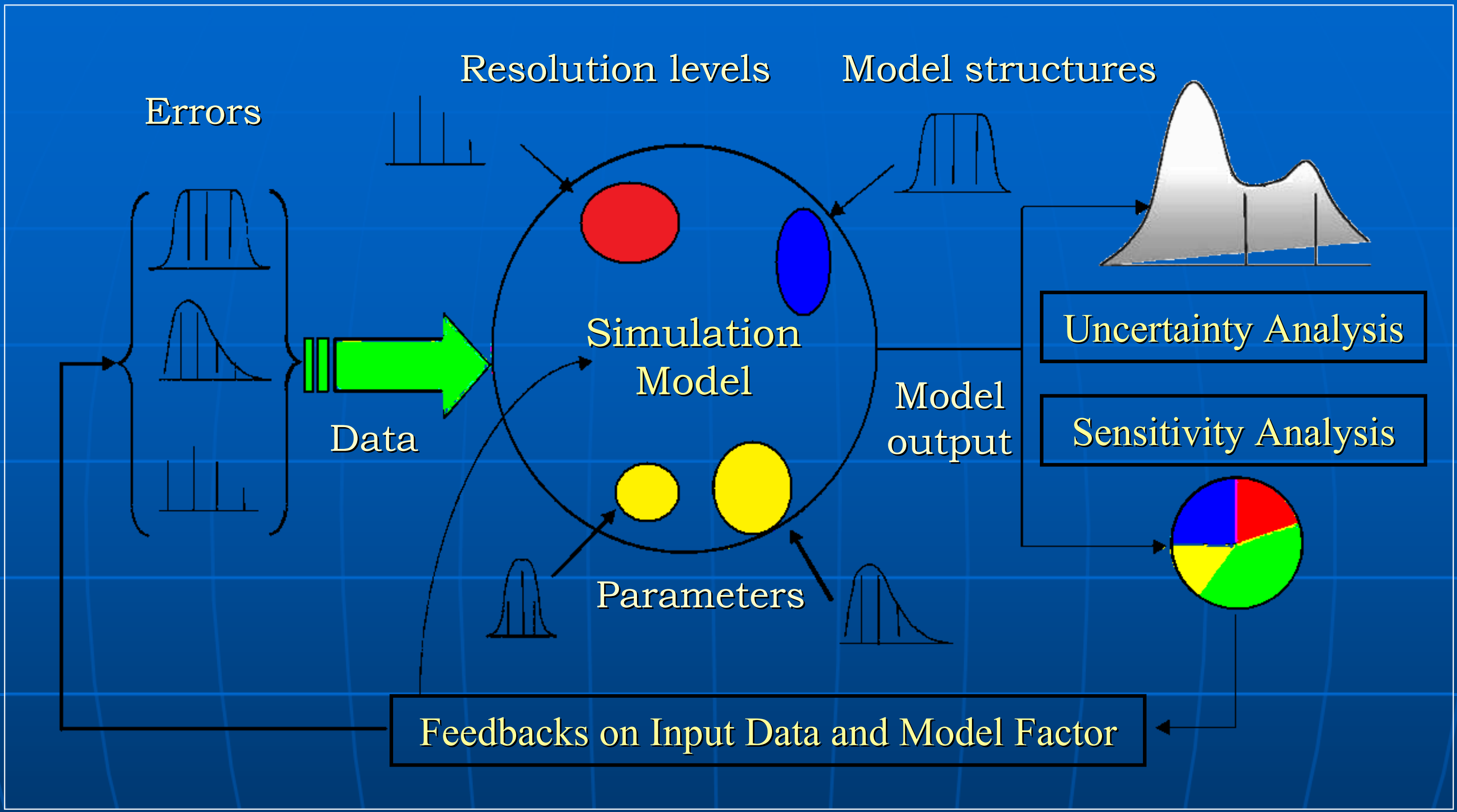
… Sensitivity analysis

Thou shall confess in the presence of sensitivity. Corollary: Thou shall anticipate criticism [···] When reporting a sensitivity analysis, researchers should explain fully their specification search so that the readers can judge for themselves how the results may have been affected. This is basically an 'honesty is the best policy' approach, advocated by Leamer, (1978[6]).

The use of mathematical modelling can be the subject of controversies, see Nassim Nicholas Taleb[7] in Economics, and Orrin H. Pilkey and Linda Pilkey Jarvis[8] in Environmental Sciences. As noted by the latter Authors, this increases the relevance of sensitivity analysis in today's modelling practice[1] .

Mathematical problems met in social, economic or natural sciences may entail the use of mathematical models, which generally do not lend themselves to a straightforward understanding of the relationship between input factors (what goes into the model) and output (the model's dependent variables). Such an appreciation, i.e. the understanding of how the model behaves in response to changes in its inputs, is of fundamental importance to ensure a correct use of the models.

- A mathematical model is defined by a series of equations, input factors, parameters, and variables aimed to characterize the process being investigated.



Ideal scheme of a possibly sampling-based sensitivity analysis. Uncertainty arising from different sources — errors in the data, parameter estimation procedure, alternative model structures — are propagated through the model for uncertainty analysis and their relative importance is quantified via sensitivity analysis.

Errors

In sensitivity analysis Type I error is assessing as important a non important factor, and Type II error assessing as non important an important factor. Type III error corresponds to analyzing the wrong problem, e.g. via an incorrect specification of the input uncertainties. Possible pitfalls in sensitivity analysis are:

- Unclear purpose of the analysis. Different statistical tests and measures are applied to the problem and different factors rankings are obtained. The test should instead be tailored to the purpose of the analysis, e.g. one uses Monte Carlo filtering if one is interested in which factors are most responsible for generating high/low values of the output.
- Too many model outputs are considered. This may be acceptable for quality assurance of sub-models but should be avoided when presenting the results of the overall analysis.
- Piecewise sensitivity. This is when one performs sensitivity analysis on one sub-model at a time. This approach is non conservative as it might overlook interactions among factors in different sub-models (Type II error).

Applications

Sensitivity analysis can be used

- To simplify models
- To investigate the robustness of the model predictions
- To play what-if analysis exploring the impact of varying input assumptions and scenarios
- As an element of quality assurance (unexpected factors sensitivities may be associated to coding errors or misspecifications).

It provides as well information on:

- Factors that mostly contribute to the output variability
- The region in the space of input factors for which the model output is either maximum or minimum or within pre-defined bounds (see Monte Carlo filtering above)
- Optimal — or instability — regions within the space of factors for use in a subsequent calibration study
- Interaction between factors

Sensitivity Analysis is common in physics and chemistry[26], in financial applications, risk analysis, signal processing, neural networks and any area where models are developed. Sensitivity analysis can also be used in model-based policy assessment studies . Sensitivity analysis can be used to assess the robustness of composite indicators [27], also known as indices, such as the Environmental Pressure Index.

Environmental

Computer environmental models are increasingly used in a wide variety of studies and applications. For example global climate model are used for both short term weather forecasts and long term climate change.

Moreover, computer models are increasingly used for environmental decision making at a local scale, for example for assessing the impact of a waste water treatment plant on a river flow, or for assessing the behavior and life length of bio-filters for contaminated waste water.

In both cases sensitivity analysis may help understanding the contribution of the various sources of uncertainty to the model output uncertainty and system performance in general. In these cases, depending on model complexity, different sampling strategies may be advisable and traditional sensitivity indexes have to be generalized to cover multivariate sensitivity analysis, heteroskedastic effects and correlated inputs.

Business

In a decision problem, the analyst may want to identify cost drivers as well as other quantities for which we need to acquire better knowledge in order to make an informed decision. On the other hand, some quantities have no influence on the predictions, so that we can save resources at no loss in accuracy by relaxing some of the conditions. See Corporate finance: Quantifying uncertainty. Sensitivity analysis can help in a variety of other circumstances which can be handled by the settings illustrated below:

- to identify critical assumptions or compare alternative model structures
- guide future data collections
- detect important criteria
- optimize the tolerance of manufactured parts in terms of the uncertainty in the parameters
- optimize resources allocation
- model simplification or model lumping, etc.

However there are also some problems associated with sensitivity analysis in the business context:

- Variables are often interdependent, which makes examining them each individually unrealistic, e.g.: changing one factor such as sales volume, will most likely affect other factors such as the selling price.
- Often the assumptions upon which the analysis is based are made by using past experience/data which may not hold in the future.
- Assigning a maximum and minimum (or optimistic and pessimistic) value is open to subjective interpretation. For instance one persons 'optimistic' forecast may be more conservative than that of another person performing a different part of the analysis. This sort of subjectivity can adversely affect the accuracy and overall objectivity of the analysis.

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Sensitivity Analysis ~ http://www.investopedia.com/terms/s/sensitivityanalysis.asp

What Does *Sensitivity Analysis* Mean?

A technique used to determine how different values of an independent variable will impact a particular dependent variable under a given set of assumptions. This technique is used within specific boundaries that will depend on one or more input variables, such as the effect that changes in interest rates will have on a bond's price.

Sensitivity analysis is a way to predict the outcome of a decision if a situation turns out to be different compared to the key prediction(s).

Investopedia explains *Sensitivity Analysis*

Sensitivity analysis is very useful when attempting to determine the impact the actual outcome of a particular variable will have if it differs from what was previously assumed. By creating a given set of scenarios, the analyst can determine how changes in one variable(s) will impact the target variable.

For example, an analyst might create a financial model that will value a company's equity (the dependent variable) given the amount of earnings per share (an independent variable) the company reports at the end of the year and the company's price-to-earnings multiple (another independent variable) at that time. The analyst can create a table of predicted price-to-earnings multiples and a corresponding value of the company's equity based on different values for each of the independent variables.

Sensitivity ~ http://www.investopedia.com/terms/s/sensitivity.asp

What Does *Sensitivity* Mean?

The magnitude of a financial instrument's reaction to changes in underlying factors. Financial instruments, such as stocks and bonds, are constantly impacted by many factors. Sensitivity accounts for all factors that impact a given instrument in a negative or positive way in an attempt to learn how much a certain factor will impact the value of a particular instrument.

Investopedia explains *Sensitivity*

Interest rates are one of the most important underlying factors in the movement of bond prices and are closely watched by bond investors. These investors get a better idea of how their bonds will be affected by interest rate movements by incorporating sensitivity into their analyses.

Spreadsheet Sensitivity Analysis

Spreadsheets and the Case Projects

The Dynamic Strategic Planning workbook is accompanied by a number of spreadsheet-based tools for data analysis. We have supplied these tools so that the users of this workbook can concentrate upon the use and implementation of decision analysis and strategic planning, rather than focusing upon the mechanics of the mathematics underlying their use.

The current form of the spreadsheets is a consequence of a combination of factors: academic research, pedagogical design, and in-class experiences. Based upon new developments, they are being routinely improved.

However, no amount of care in tool design can substitute for expertise on the part of the user.

The case projects have been designed assuming that these tools will be used effectively. The purpose of this document is to assure that you, the user of these tools, are prepared to exploit them to their fullest - specifically, that you are able to make use of spreadsheet sensitivity analysis tools.

http://msl1.mit.edu/rdn/d_table.pdf

Sensitivity Analysis Using Excel

The main goal of sensitivity analysis is to gain insight into which assumptions are critical, i.e., which assumptions affect choice. The process involves various ways of changing input values of the model to see the effect on the output value. In some decision situations you can use a single model to investigate several alternatives. In other cases, you may use a separate spreadsheet model for each alternative.

MANUAL WHAT-IF ANALYSIS

Using this approach, you enter values into cells C4:C6 and see what the effect is on net cash flow.

For example, with the predetermined price of $29, you may think that Units Sold will be in the range between 500 and 900 units. Keeping other input assumptions at base case, the corresponding Net Cash Flows are $–1,500 and $6,900. When we vary a single input assumption, keeping all other input assumptions at their base case values, we say we are doing "one at a time" or "singlefactor“ sensitivity analysis.

	A	B	C
1	Controllable Input		
2		Unit Price	$29
3	Uncontrollable Inputs		
4		Units Sold	700
5		Unit Variable Cost	$8
6		Fixed Costs	$12,000
7	Performance Measure		
8		Net Cash Flow	$2,700

http://www.treeplan.com/chapters/02_decan_20071029_1042.pdf

Chapter 6: **Sensitivity Analysis**

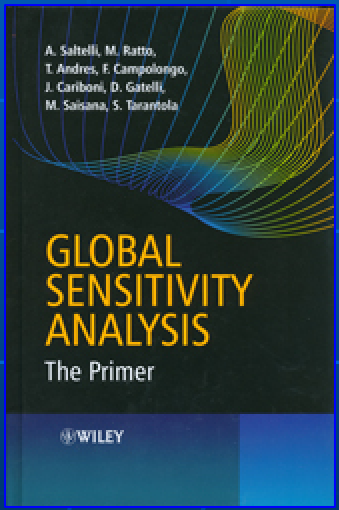
Suppose that you have just completed a linear programming solution witch will have a major impact on your company, such as determining how much to increase the overall production capacity, and are about to present the results to the board of directions. How confident are you in the result? How much will the results change if your basic data (e.g. profit per item produced, or availability of a component) is slightly wrong? Will that have a minor impact on your result? Will it give a completely different outcome, or change the outcome only slightly?

These are the kinds of questions addressed by sensitivity analysis. Formally, the question is this: is my optimum solution (both the values of the variables and the value of the objective function) sensitive to a small change in one of the original problem coefficients (e.g. coefficients of the variables in the objective function or constrains, or the right hand side constants in the constraints)? If Z or x_i change when an original coefficient is changed, then we say that the LP is sensitive. We could ask, for example, if the Acme Bicycle Company solution is sensitive to a reduction in the availability of the metal finishing machine from 4 hours per day to only 3 (i.e. a change in the third constraint from $x_1+x_2\leq 4$ to $x_1+x_3\leq 3$).

http://www.sce.carleton.ca/faculty/chinneck/po/Chapter6.pdf

Sensitivity Analysis

Global Sensitivity AnalysisThe Primer
Saltelli, A., Ratto, M., Andres, T., Campolongo, F., Cariboni, J., Gatelli, D. Saisana, M., and Tarantola, S., 2008, John Wiley & Sons (ISBN: 978-0-470-05997-5)**Who needs** Sensitivity Analysis



Tutorial on Sensitivity Analysis

SimLab Software for Sensitivity Analysis

What's New

•Sixth **International Conference**on **Sensitivity Analysis of Model Output**, Bocconi University of Milan, **19-22 July 2010**

•Sixth **Summer School** on**Sensitivity Analysis of Model Output**, Villa La Stella, Fiesole - Florence, **14-17 September 2010**

http://sensitivity-analysis.jrc.ec.europa.eu/

Sensitivity Analysis

CA∫DE

Multi-disciplinary Design Optimization

Centre for Aerospace Systems Design & Engineering
Department of Aerospace Engineering
Indian Institute of Technology
Mumbai 400 076

sensitivity.ppt (67 slides)

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